Undergraduate Research and the Field of Mathematics

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Mathematics is a very old and important part of human culture. We find its origins in the human attempts to quantify the laws of nature. It is tempting to think of modern mathematics as a cult of truth, founded by Isaac Newton three hundred years ago. Whatever your views of this subject, it is easy to recognize some of its features: abstractness, precision, rigor, indisputable character of its conclusions, and finally, the exceptionally broad range of its applications. One of the most common questions I am asked by people when they learn that I am a research mathematician is "What do people research in Mathematics?". They usually go on to state that in mathematics everything is already known since we know that two plus two is four. Much like in any other discipline mathematical research is work which is undertaken systematically to increase the stock of knowledge by discovering new theorems or proving existing hypothesis, reaffirm the results of previous work, solve new or existing problems, or develop new theories.

The principle reason that people and students in particular have such a hard time contemplating that somebody can discover something new in mathematics is that their knowledge limits their ability to ask really hard open questions. Unlike in disciplines like Astronomy where even major discoveries are made by amateurs, mathematics require substantial familiarity with the existing body of knowledge before the research can take the place.

Unlike a biology research project that might help discover a critically-needed vaccine the implications of theoretical mathematical research on our everyday's life appears to be less obvious. Nothing could be further from the truth as every time you use your cell phone, turn on your mp3 player, watch a movie, pull your credit card to pay a bill, or even get a flu vaccine you are just enjoying the fruits of theoretical mathematical research.

Upon my arrival to former Augusta State University, I took a formidable task of trying to engage undergraduate students in mathematics research. My previous experience consisted of mentoring a small team of students during the 2007 Arizona Summer Program on Mathematical Modeling, an NSF sponsored 4-week research experience for undergraduates at the University of Arizona. Most participants in the Arizona program were graduating seniors who were already admitted into graduate programs or were contemplating going to graduate school. As my audience in Augusta was very different I decided to have another approach.

It is very important for any advisor engaged in undergraduate research and undergraduate research in mathematics in particular, not to underestimate the potential for original undergraduate research. It is all too easy to create failure by expecting failure than it is to give students an exaggerated view of their own abilities. Mathematics research is difficult, but there's no reason to make it even more difficult by creating an atmosphere where failure is the expected norm. In my point of view ability to think clearly and originally is far more import-

ant in doing research than factual knowledge and familiarity with current literature. My selection process is long and starts early in the fall semester when I ask interested students to sign up for the Putnam Mathematical Competition. The Putnam competition is the annual mathematics competition for undergraduate college students in the United States and Canada and is held on the first Saturday in December. The problems are very deep and require ingenuity rather then applications of learned facts. During the Putnam preparation season preceding the competition I meet with students once a week for two hours to work on problems. We usually sit quietly together around the table and work individually until somebody makes progress on a problem. At that point the person who thinks that she/he made progress tries to convince the other team members in the correctness of her/his solution and write up the proof. It should come as a no surprise that I am not always the first one who makes a progress. Seeing me struggling with problems is usually a very humbling experience for students. Sometimes our sessions end up as a lectures initiated by students question about unfamiliar notions in their attempt to understand a question.

The attrition rate during the preparation season is very high and taking a Putnam test in December is usually a very strong indication of exceptional mathematics aptitude if not ability. The actual research starts in the spring semester following Putnam preparation season. It starts by me asking students to read a recent research article and propose a question worth investigating. The amount of knowledge required to understand recent progress usually far exceeds student familiarity with the subject and requires from them to become squinted with new mathematical ideas on their own in very rapid fashion. It is of paramount importance at this stage not to forget that the goal of doing mathematics research is not learning the subject but rather discovering new things. This puts a great responsibility on the mentor on one hand in assigning student doable problems and on the another hand preventing them from just studying. It is often the case that the problems I assign students lead to no results. Sometimes after a few weeks of hard work, a student makes no significant progress.

In these situations I give a new problem to the student. Matching students and problems is a tricky process that requires a lot of educated guessing tempered by experience which I have not quite mastered yet. Finding suitable problems is an even greater challenge and I am on the constant lookout while reading peer reviewed journals, talking to experts in the field, and leaders of better known REU programs. Creating a local database of suitable papers/problems would be a great addition to my current GRU effort. It is absolutely mandatory that students write their work in a format suitable for publication in a research journal. If you don't write up your results, it's just as if you didn't even get them at all. Writing for publication is an important aspect of being a professional mathematician. The publication process can be a very intimidating experience, even for experienced mathematicians. Without help, most students would not even realize that their work is of professional caliber.

At the end of the spring program students are also required to give a presentation on their research to other interested students and faculty.