Math 3280 Linear Algebra
Section A

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Final Exam

Student Name:__________________
Student ID#:__________________

Each problem is worth 10 points. Give a complete solution to receive the full credit!

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1. A complex number is an expression of the form $x + yi$ where x and y are real numbers and i is the imaginary unit, satisfying $i^2 = -1$. One defines addition of complex numbers as

$$(x_1 + y_1 i) + (x_2 + y_2 i) = (x_1 + x_2) + (y_1 + y_2)i$$

and multiplication of complex numbers by real numbers as

$$\lambda(x_1 + y_1 i) = \lambda x_1 + \lambda y_1 i.$$

Prove that the set of all complex numbers denoted as $\mathbb{C}$ is a vector space over the field of real numbers $\mathbb{R}$

2. Show that the complex numbers $1+2i$ and $2-i$ form a base for the vector space $\mathbb{C}$ of all complex numbers.

3. The complex conjugate of a complex number $z = x + yi$ is defined to be

$$\bar{z} = x - yi.$$

It is trivial to prove that complex conjugation is a linear mapping of $\mathbb{C}$ into itself i.e.

$$\overline{(x_1 + y_1 i)} + \overline{(x_2 + y_2 i)} = \overline{(x_1 + y_1) + (x_2 + y_2) i}$$
$$\overline{\lambda(x_1 + y_1 i)} = \lambda\ \overline{(x_1 + y_1 i)}$$

Give a matrix representation of the conjugation of complex numbers with respect to the standard base of $\langle (1+0i), (0+1i) \rangle$ of $\mathbb{C}$.

4. Compute the row reduced echelon form of the matrix

$$\left(\begin{array}{rr|rr} 1 & 2 & 1 & 2 \\ 2 & -1 & -2 & 1 \end{array}\right).$$

Give the matrix representation of the conjugation of complex numbers with respect to the standard base of $\langle (1+2i), (2-1i) \rangle$ of $\mathbb{C}$.

5. Show that conjugation of complex numbers is an automorphism of the vector space $\mathbb{C}$. [Hint: What is the Kernel space of the conjugation?]

Definition 1 *The matrices T and S are similar if there is a nonsingular matrix P such that $T = PSP^{-1}$.*

6. Prove that similarity preserves determinants.

Definition 2 *An endomorphism/automorphism of vector spaces is diagonalizable if it has a diagonal representation.*

7. Diagonalize

$$A = \begin{pmatrix} -0.6 & 0.8 \\ 0.8 & 0.6 \end{pmatrix}$$

and then compute A^{2012}.

8. Find the characteristic polynomial and the eigenvalues of the matrix.

$$M = \begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix}$$

Theorem 1 (Cayley-Hamilton) *The minimal polynomial of an endomorphism divides characteristic polynomial of the same endomorphism.*

9. Find the minimal polynomial of the matrix.

$$M = \begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix}$$

10. Find the Jordan form and a Jordan basis of the matrix

$$M = \begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix}$$