Exam 2

Student Name:__________________
Student ID#:__________________

Each problem is worth 5 point. Give a complete solution to receive the full credit!

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1. Draw a Venn diagram for $U = \{1, 2, 3, 4, 5\}$, $A = \{1, 3, 4\}$, $B = \{3, 4, 5\}$, and $C = \{2, 4\}$. Are sets B and $U \setminus A$ disjoint sets?

2. List partitions of the set $\{1, 2\} \times \{a\}$.

3. Let $x \in \mathbb{R}$. The *floor* function of x, denoted by $f(x) = \lfloor x \rfloor$, is the largest integer less than or equal to x. Show that $(f \circ f^{-1})(\{0, 1\}) = \{0, 1\}$. Is the *floor* function onto function from the set of real numbers into the set of integers?

4. Consider two permutations $f = (2, 1, 4, 3, 5)$ (which is given in one line form) and $g = (1, 3)(5, 2, 4)$ (which is given in cycle form) of the set $A = \{1, 2, 3, 4, 5\}$. Find permutation $h = (f^{-1} \circ g) \circ g$ of the set A.

5. Evaluate $\phi(105)$ where $\phi(x)$ is the Euler function.

6. There were N students in a class. Their exam scores ranged between 27 and 94. All possible scores were achived by at least one student except for the scores 31, 42, and 56 (none of the students got those scores). What is the smallest value of N that guarantees that at least four students achived the same score?

7. If we compute 11^t % 163 for $t = 0, 2, \ldots, 161$ we get each of the numbers $1, 2, \ldots, 162$ exactly once. Solve the equation $41 = 11^t$ % 163 using your favorite programming language.

8. Using the Euclidean algorithm and your favorite programming language, find A and B such that $Am + Bn = \gcd(m, n)$ where $m = 1012$ and $n = 54$. How many common divisors are there of $m = 1012$ and $n = 54$? List all common divisors of $m = 1012$ and $n = 54$.

9. Evaluate Stirling number $S(7,3)$. Let now $A = \{a,b,c,d,e,f,g\}$. How many *onto* functions are there from the set A to the set $B = \{1,2,3\}$?

10. Let R be a relation of length two on the set $\mathbb{N}_0 \times \mathbb{N}_0$ defined as $(a,b)R(m,n)$ if and only if $a+n = m+b$. Show that relation R is an equivalent relation. How many equivalence classes does it have?