Final Exam

Student Name:__________________
Student ID#:__________________

Each problem is worth 10 points. Give a complete solution to receive the full credit!

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1. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{a, b, c, d\}$. Consider the function $f : A \to B$ given as

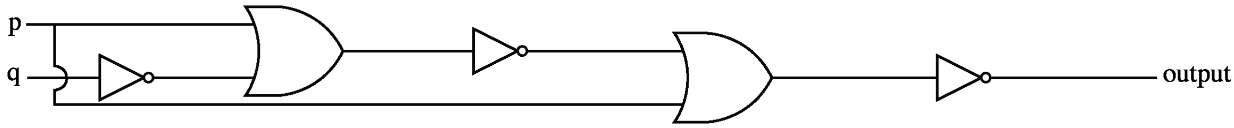
$$f = \{(1, a), (2, b), (3, d), (4, b), (5, c)\}$$

Compute set $(f \circ f^{-1}) (\{a, b, c\} \setminus \{c, d\})$.

2. Let $S = \mathcal{P}(T)$, be the power set of $T = \{2, 3, 4\}$. How many binary relations on S are reflexive? Explain your reasoning!

3. Let $S = \mathcal{P}(T)$, be the power set of $T = \{a, b, c\}$. Define a binary relation on S by $X \; R \, Y$ if either $X \subseteq Y$ or $Y \subseteq X$. Show that relation R is not an equivalence relation.

4. Write a logic statement corresponding to the following logic circuits. Construct a truth table showing the output of the circuit.



5. A maximal element of a subset S of some partially ordered set is an element of S that is not smaller than any other element in S. The notion of a maximal element is weaker than that of the greatest element. The greatest element of a subset S of a partially ordered set is an element of S which is larger than or equal to any other element of S. Let
$$S = \{\{d, o\}, \{d, o, g\}, \{g, o, a, d\}, \{o, a, f\}, \{g, o, a, f\}\}$$
be ordered by containment ($\subseteq$). Find maximal element(s) of S if there is any. Find the greatest element of S if there is any.

6. Construct the logical circuit for the Boolean function $S(p, q, r)$ given by the following table.

p	q	r	$S(p,q,r)$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

7. Using binary arithmetics, a number y is computed by taking the 8-bit two's complement of $x - c$. If $x = 11001001_2$ and $c = 10101_2$ then $y =$?

8. Define $f(n) = \lfloor \frac{2n+1}{2} + \frac{1-(-1)^{2n+1}}{4} \rfloor$ for all $n \in \mathbb{Z}$. Thus, $f : \mathbb{Z} \to \mathbb{Z}$, where $\mathbb{Z}$ is the set of all integers. Find $\mathrm{Im}(f)$.

9. A computer science student is studying under a tree and another pulls up on a flashy new bike. The first student asks, "Where'd you get that?" The student on the bike replies, "While I was studying outside, a beautiful girl pulled up on her bike. She took off all her clothes and said, 'You can have anything you want'." The first student responds, "Good choice! Her clothes probably wouldn't have fit you."

   How many computer science students in above story are female?

10. A set of three positive integers (a, b, c) is a Pythagorean triple if they satisfy the Pythagorean relation $a^2 + b^2 = c^2$. If the greatest common divisor of the three integers is 1, that is, $gcd(a, b, c) = 1$, then the Pythagorean triple is called primitive. For example, $(3, 4, 5)$ is a primitive Pythagorean triple and $(6, 8, 10)$ is a Pythagorean triple that is **not** primitive.

    (a) Show that if (a, b, c) is a Pythagorean triple, then (ka, kb, kc) is a Pythagorean triple for all integers k greater than 1.

    (b) Show that if a is an integer and a^2 is an even integer, then a is an even integer.

    (c) Using parts (a) and (b) and by considering different cases, depending on whether a and b are odd or even integers, show that if (a, b, c) is a primitive Pythagorean triple, then exactly one of a, b, and c is an even integer.