Exam 1

Student Name:____________________
Student ID#:____________________

Each problem is worth 5 points. Give a complete solution to receive the full credit!

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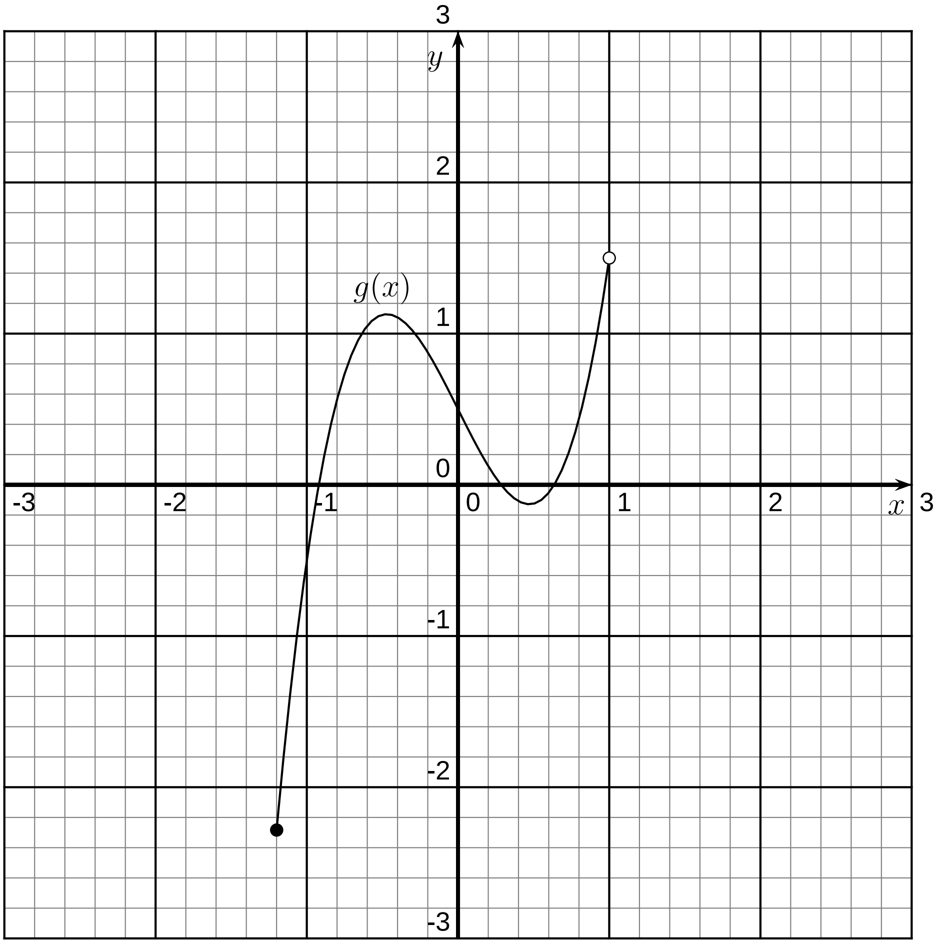
1. Replace the question mark by $<$, $>$, or $=$, whichever is correct.
   (a) $\frac{1}{2}$? $\sin\left(\frac{\pi}{6}\right)$
   (b) $\frac{1}{3}$? 0.33333333333
   (c) $\sqrt[6]{2}$? $\sqrt[3]{\frac{\sqrt{18}}{3}}$
   (d) 2 ? $e^{\ln 2}$
   (e) π ? $\frac{22}{7}$

2. Consider the table giving values for height and weight of 5 individuals. Determine which

Height	61	63	63	66	64
Weight	146	174	123	126	138

   of the following best describes the relationship between height and weight.
   (a) Height is a function of weight.
   (b) Weight is a function of height.
   (c) Height is a function of weight and weight is a function of height.
   (d) None of the above.

3. The graph of the function $g(x)$ is given.



(a) State approximately the domain of the function.

(b) State approximately the interval(s) on which $g(x)$ is decreasing.

(c) State approximately $\lim_{x\to 1} g(x)$.

4. Give an example of a infinite, bounded above, monotone increasing sequence. What is the fifth term of your sequence?

5. Decide whether the sequence $a_n = \sqrt[3]{\frac{27n^2+n-6}{n^2+16}}$, $n = 1, 2, 3, ...$ converges or diverges. If the sequence converges, find its limit.

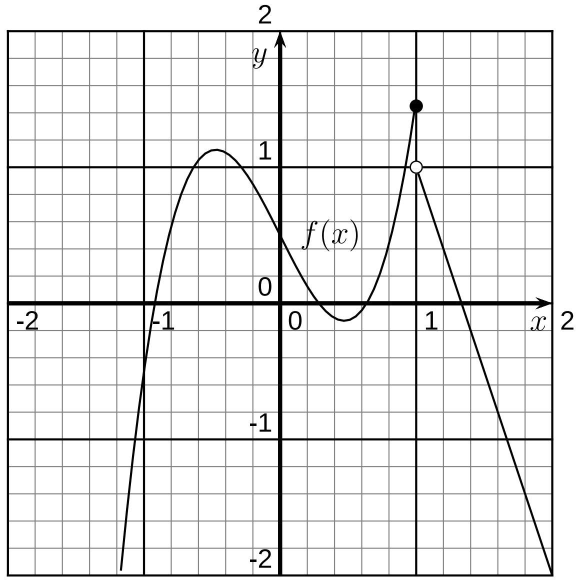
6. The function f is defined by

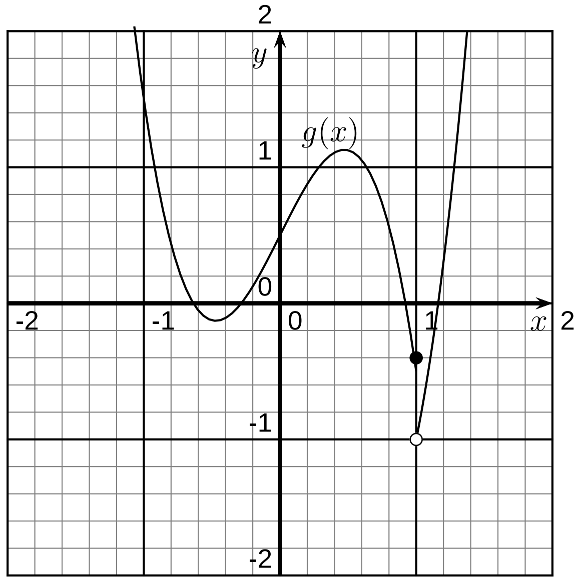
$$f(x) = \begin{cases} 3x - 1, & -3 \le x < 0 \\ x^2 - 5, & 0 \le x < 2 \\ 1 - x, & 2 \le x \le 4 \end{cases}$$

Evaluate the limits $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$. Is the function continuous at the point $x = 2$?

7. Let $A = (0, 1) \subseteq \mathbb{R}$. Is A a bounded set of the set of real numbers? Give a lower and an upper bound for A. What is the greatest lower bound for A (also known as the infimum)? What is the least upper boud for A (also known as the supremum)?

8. The graphs of f and g are given.





Use them to evaluate $\lim_{x\to 1^-} (\frac{f}{g})(x))$ if it exists.

9. Find the

$$\lim_{x\to-\infty} \tan^{-1}(x).$$

10. **Theorem 1 (The Bolzano-Cauchy intermediate-value theorem)** *If a function that is continuous on a closed interval assumes values with different signs at the endpoints of the interval, then there is a point in the interval where it assumes the value 0.*

    Use the Bolzano-Cauchy intermediate-value theorem to show that the equation

$$\cos(x) - \frac{1}{2}e^x = 0$$

    has a solution on $[-\pi, 0]$.