Exam 2

Student Name:____________________
Student ID#:____________________

Each problem is worth 5 points. Give a complete solution to receive the full credit!

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1. Decide whether the sequence $a_n = \cos(\pi n),\ n = 1, 2, 3, ...$ converges or diverges. If the sequence converges, find its limit.

2. The function f is defined by

$$f(x) = \begin{cases} x^2 - a, & 0 \le x < 2 \\ 7 - 2x, & 2 \le x \le 4 \end{cases}$$

where a is a parameter. Find its value so that the function is continuous at the point $x = 2$.

3. Using the definition, find the derivative of the function $h(t) = \sqrt{t+1}$ at the point $t = 1$.

4. Find the best affine approximation of the function $g(z) = \log_3 z$ at the point $z = 5$. Use it to approximate $\log_3 5.2$. What is the difference between the approximate value and the "*true*" value obtained by a calculator?

5. Determine the domain of the function $f(x) = \sqrt{(x-5)^2}$ and then find the points at which the function fails to be differentiable. Is the function continuous in those points?

6. Find the derivatives of the following functions.
   (a) $f(x) = \left(\frac{1}{x} - 3\right)\frac{x^2+3}{2x-1}$
   (b) $g(x) = 2^x - \log_5(x) + 5\tan(x)$

7. Find the derivatives of the following functions.
   (a) $f(x) = \left(\frac{1}{x} - 3\right)^3$
   (b) $g(x) = 2^{-\sin(x)} - \log_5(\tan(x))$

8. Find the coordinates of the two points on the closed curve $x^2 - 2x + 4y^2 + 16y + 1 = 0$ where the line tangent to the curve is vertical.

9. Let $g(x)$ be a function defined as $g(x) = \frac{4x}{1+4x^2}$ for all $x > 0$. Find the absolute maximum value of g on the open interval $(0, \infty)$ if the maximum exists. Find the point(s) in which absolute minimum value of g is attained on the open interval $(0, \infty)$ if the minimum exists.

10. **Theorem 1 (Lagrange's finite-increment theorem)** . *If a function* $f : [a, b] \to \mathbb{R}$ *is continuous on a closed interval* $[a, b]$ *and differentiable on the open interval* (a, b), *there exists a point* $\xi \in (a, b)$ *such that*

$$f(b) - f(a) = f'(\xi)(b - a).$$

Use Lagrange's finite-increment theorem to solve the following problem. Suppose that we know that $f(x)$ is continuous and differentiable on $[6, 15]$. Let's also suppose that we know that $f(6) = -2$ and that we know that $f'(x) \leq 10$. What is the largest possible value for $f(15)$?