Exam 3

Student Name:__________________

Student ID#:__________________

Each problem is worth 5 points. Give a complete solution to receive the full credit!

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1. Evaluate $\displaystyle\int \left(3z - \frac{4}{z^2}\right) dz$.

2. Evaluate $\displaystyle\int \left(\cos(\theta) - \frac{2}{\theta} + 2^\theta\right) d\theta$.

3. Evaluate $\int \left(\cos(\theta + 3) - 2^{\theta+5}\right) d\theta$.

4. Evaluate $\int \theta^2 \sin(\theta)\ d\theta$.

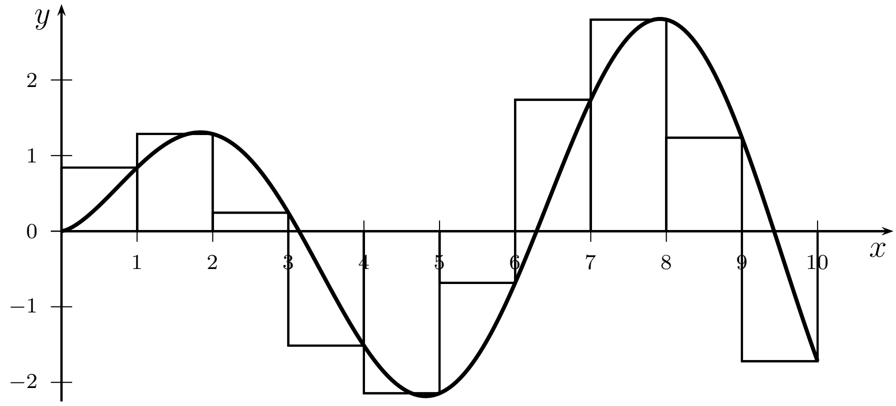
5. Evaluate $\int_{\pi}^{\pi} \sin^6(x)\cos^6(x)\ dx$.

6. Evaluate the integral $\int_{-1}^{2} (x - 8|x|)\, dx$.

7. Find the right hand Riemann sum that approximates the area under the curve

$$f(x) = \sqrt{x}\sin(x)$$

and above the interval $[0, 10]$ as shown in the figure.

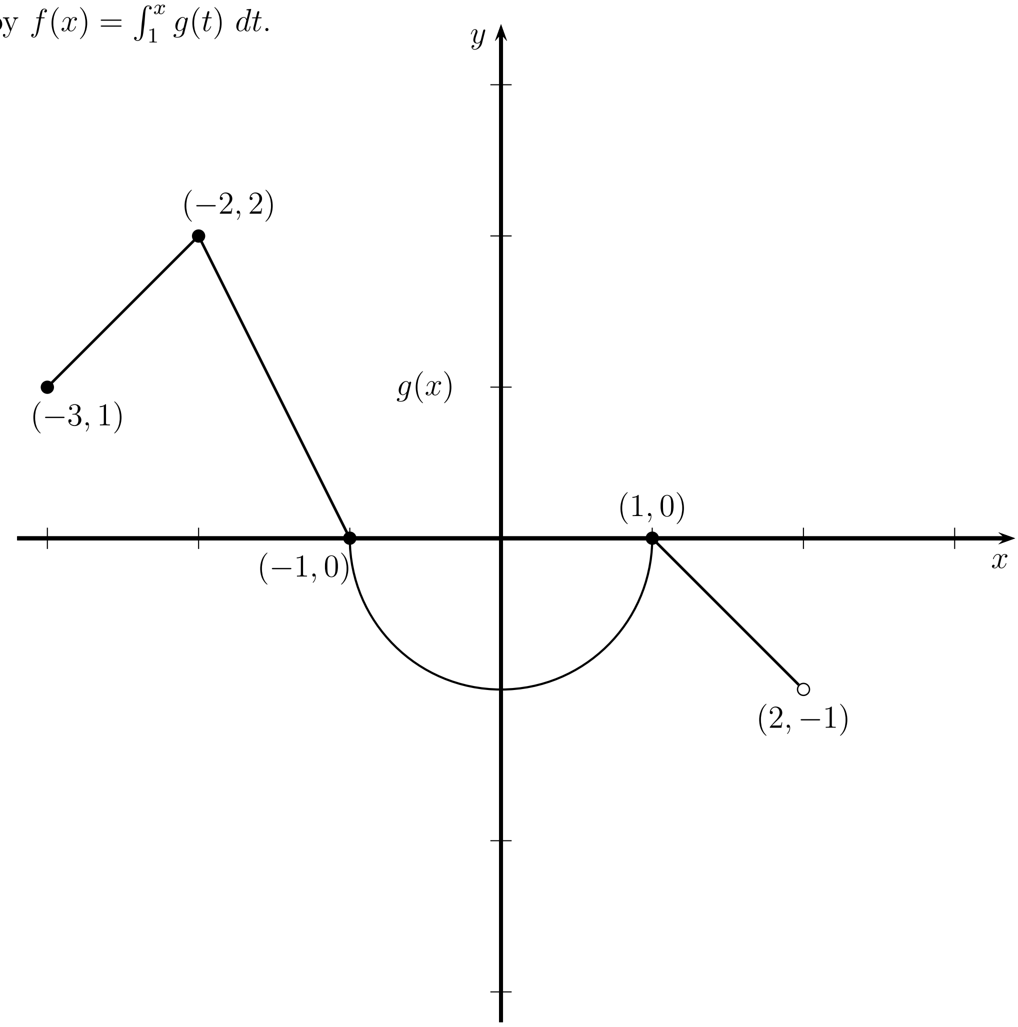


8. Let $f(x)$ be the characteristic function of the set $\mathbb{Q}$ of rational numbers restricted to the closed interval $[0, 1]$.

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$$

Let $\mathcal{P} = (0, 0.5, 1)$ be a regular partition of $[0, 1]$. Write the upper $U_{f,\mathcal{P}}$ and lower $L_{f,\mathcal{P}}$ Darboux sum of function $f(x)$ with respect to $\mathcal{P}$. Evaluate $U_{f,\mathcal{P}} - L_{f,\mathcal{P}}$. Let $\mathcal{P}' = (0, 0.25, 0.5, 0.75, 1)$ be a refinement of partition $\mathcal{P}$ obtained by cutting the subintervals into smaller pieces. Write the upper $U_{f,\mathcal{P}'}$ and lower $L_{f,\mathcal{P}'}$ Darboux sum. Evaluate $U_{f,\mathcal{P}'} - L_{f,\mathcal{P}'}$.

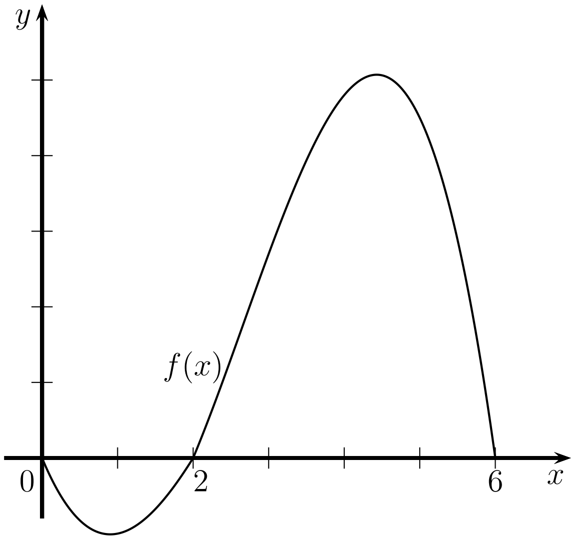
9. Let g be the continuous function defined on $[3, 2)$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given below. Let f be the function given by $f(x) = \int_1^x g(t)\ dt$.



(a) Find the values of $f(2)$ and $f(-2)$.

(b) For each of $f'(-1)$ and $f''(-1)$, find the value or state that it does not exist.

10. The graph of the differentiable function $y = f(x)$ with domain $0 \le x \le 6$ is shown on the figure. The area of the region enclosed between the graph of f and the x-axis for $0 \le x \le 2$ is 1.8, and the area of the region enclosed between the graph of f and the x-axis for $2 \le x \le 6$ is 11.



(a) Evaluate $\displaystyle\int_0^6 (3f(x) + 2)\,dx$. Show the computations that lead to your answer.

(b) Let $g(x) = \displaystyle\int_2^x f(t)\,dt$. On what intervals, if any, is the graph of g both concave up and decreasing? Explain your reasoning.