

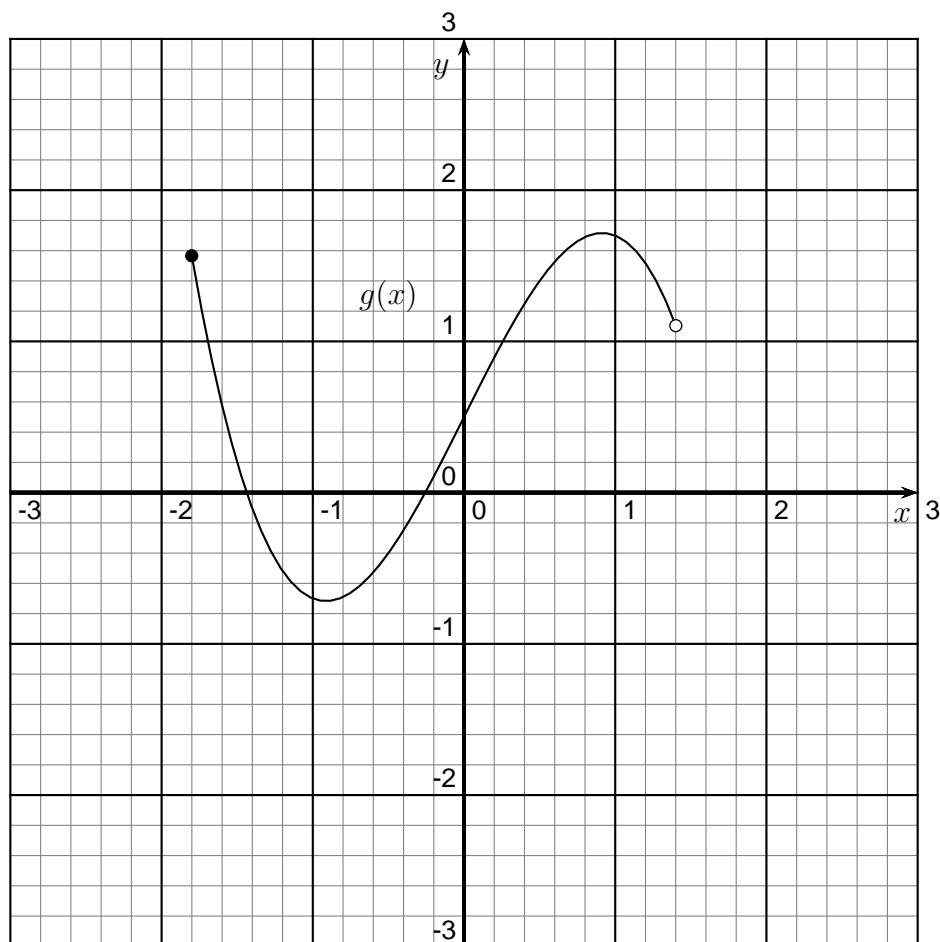
Final Exam

Student Name: _____

Student ID#: _____

Each problem is worth 10 points. Give a complete solution to receive the full credit!

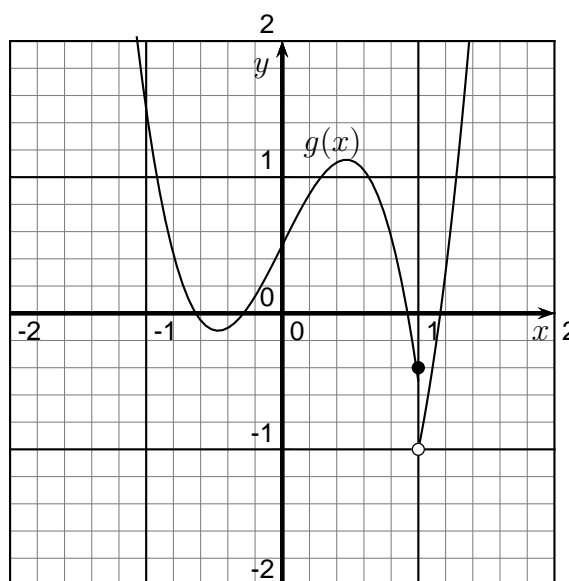
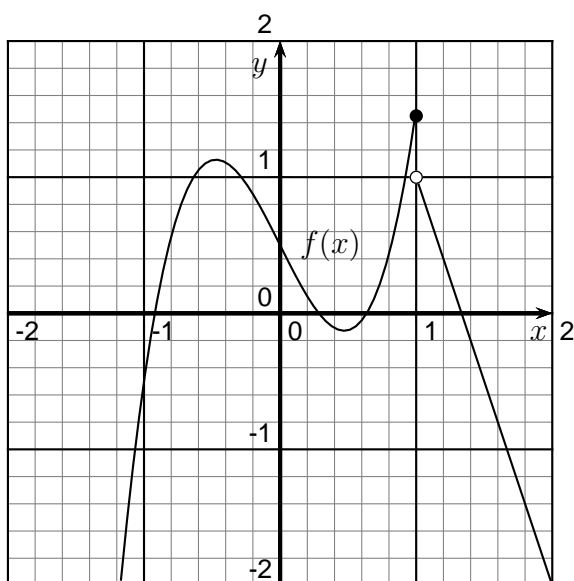
1. The graph of the function $g(x)$ is given.



- State approximately the domain of the function.
- State approximately the interval(s) on which $g(x)$ is decreasing.
- State approximately critical points.

2. Write 5th term of the sequence $a_n = \frac{27n^2+n-6}{n^2+16}$, $n = 1, 2, 3, \dots$. Decide if the sequence is converges or diverges. If the sequence converges decide if it is bounded or unbounded.

3. The graphs of f and g are given.



Use them to evaluate $\lim_{x \rightarrow 1^+} (f + g)(x)$ if it exists.

4. The function f is defined by

$$f(x) = \begin{cases} \cosh x - a, & -2 \leq x < 0 \\ 7 - 2x, & 0 \leq x \leq 4 \end{cases}$$

where a is a parameter. Find its value so that the function is continuous at the point $x = 0$.

5. Find the best affine approximation of the function $g(z) = \sqrt[3]{1+z}$ at the point $z = 0$. Use it to approximate $\sqrt[3]{1.1}$. What is the difference between the approximate value and the “*true*” value obtained by a calculator?

6. Find the coordinates of the two points on the closed curve $x^2 - 2x + 4y^2 + 16y + 1 = 0$ where the line tangent to the curve is perpendicular to the line $y = \frac{1}{2}x - 3$

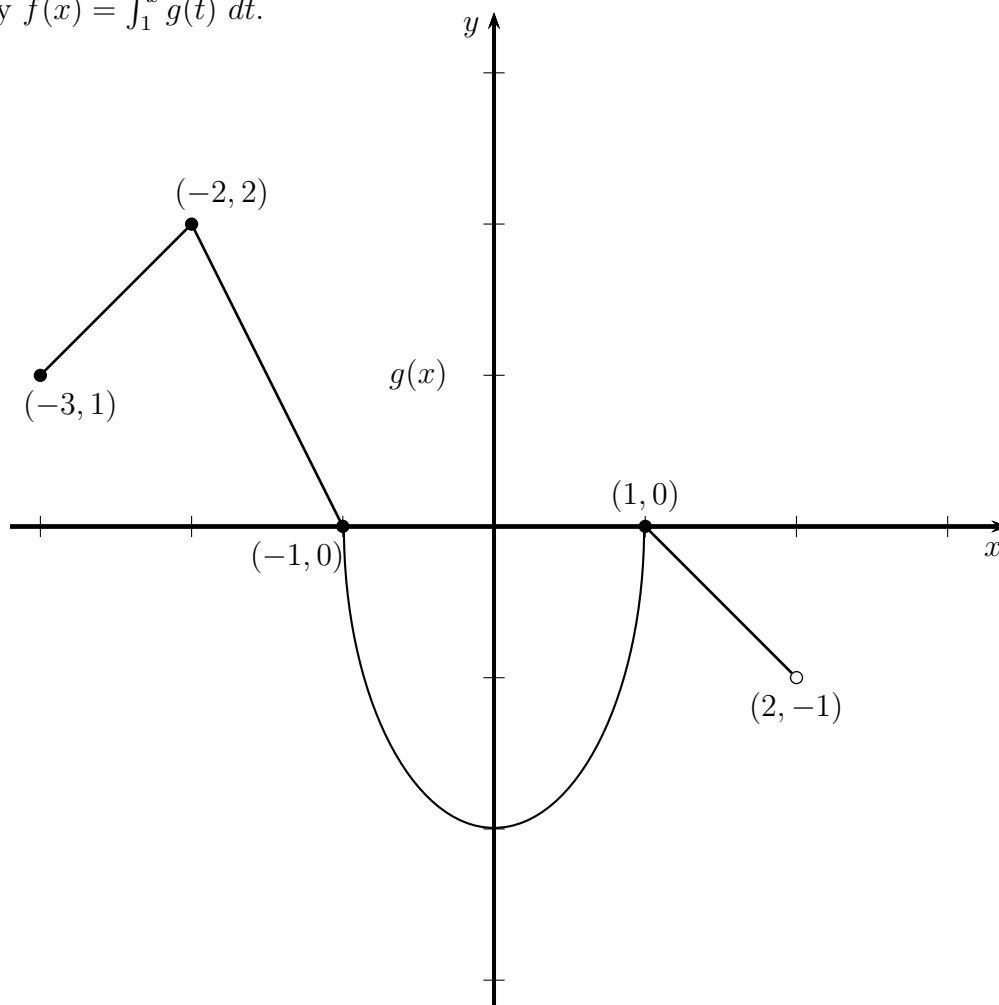
7. Evaluate the integral $\int_{-1}^2 \left(\frac{1}{1+x^2} - 8\sqrt{x^2} \right) dx$.

8. Evaluate

(a) $\int (1 + \tan \theta)^6 \sec^2 \theta \, d\theta.$

(b) $\int \theta^2 \sinh(\theta) \, d\theta.$

9. Let g be the continuous function defined on $[-3, 2)$ whose graph, consisting of three line segments and a semiellipse centered at the origin, is given below. Let f be the function given by $f(x) = \int_1^x g(t) dt$.



- (a) Find the values of $f(2)$ and $f(-2)$.
- (b) For each of $f'(-1)$ and $f''(-1)$, find the value or state that it does not exist.

10. Find the limit.

$$\lim_{x \rightarrow \infty} \frac{\cosh(x)}{e^x}$$