Exam 1

Student Name:_____
Student ID#:_____

Each problem is worth 5 points. Give a complete solution to receive the full credit!

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1. Determine whether the sequence $a_n = \ln(n+1) - \ln(n)$ converges or diverges. If it converges, find its limit.

2. Write 5^{th} term of the sequence $a_n = \frac{\arctan(n)}{n}$, $n = 1, 2, 3, ...$ Determine whether the sequence is increasing, decreasing, or not monotonic. Is the sequence bounded?

3. What is the sum of the infinite geometric series $2 + 0.5 + 0.125 + 0.03125 + \ldots$?

4. Investigate the behavior (convergence or divergence) of $\sum_{n=1}^{\infty} a_n$ if $a_n = \dfrac{\sqrt{n+1} - \sqrt{n}}{n}$.

5. Decide if the series $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$ is convergent or divergent by telescoping.

6. Show that if $a_n > 0$ and $\sum_{n=1}^{\infty} a_n$ is convergent, then $\sum_{n=1}^{\infty} \ln(1 + a_n)$ is also convergent.

7. Determine if the series $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^2}$ is absolutely convergent, conditionally convergent, or divergent.

8. Evaluate indefinite integral $\int \frac{\cos(\pi/x)}{x^2}\, dx$.

9. Evaluate $\displaystyle\int_1^\infty \frac{\ln(x)}{x^p}\,dx$ for $p > 1$.

10. Find the derivatives of the following functions.

(a) $f(x) = \left(\log_5\left(\arctan(x)\right) + 2^x\right)^3$

(b) $g(x) = 2^{-\cosh(x)} - \log_5(\frac{1}{x})$