Exam 2

Student Name:__________________
Student ID#:__________________

Each problem is worth 5 point. Give a complete solution to receive the full credit!

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1. Find the domain of the Bessel function of order 0 defined by

$$J_0 = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$$

2. Show that J_0 (the Bessel function of order 0) satisfies the differential equation

$$x^2 J_0''(x) + xJ_0'(x) + x^2 J_0 = 0$$

3. Evaluate $\int_0^1 J_0(x)dx$ to two decimal places. Note that $J_0(x)$ stands for the Bessel function of order 0 as before.

4. Evaluate the indefinite integral $\int \frac{e^x - 1}{x} dx$ as an infinite series.

5. Evaluate the limit.

$$\lim_{x \to 0} \frac{\tan(x) - x}{x^3}$$

6. Use division of power series to find the first three nonzero terms in the Maclaurin series for $y = \sec(x)$.

7. Use the series to approximate $\frac{1}{\sqrt{2\pi}}\int_0^1 e^{-\frac{x^2}{2}}dx$ to two decimal places. In statistics the 68-95-99.7 rule or three-sigma rule states that for a normal distribution, nearly all values lie within 3 standard deviations of the mean. According to that rule if the random variable $X \sim N(0,1)$ then $P(-1 < X < 1) = \frac{1}{\sqrt{2\pi}}\int_{-1}^1 e^{-\frac{x^2}{2}}dx \approx 0.68$. Use your computation to prove three-sigma rule for the case of 1 standard deviation.

8. Evaluate indefinite integral $\int e^t \sin(t)\ dt$.

9. Evaluate the integral if it is convergent.

$$\int_0^1 \frac{\ln(x)}{\sqrt{x}}\ dx$$

10. Find the derivatives of the following functions.

(a) $f(x) = \left(\log_5\left(\arcsin(x)\right) + \int_2^x e^{-t^2}\ dt\right)^3$

(b) $g(x) = 2^{-\log_5(\frac{1}{x})} + \int_1^{x^2} \sin(\ln(t))\ dt$