

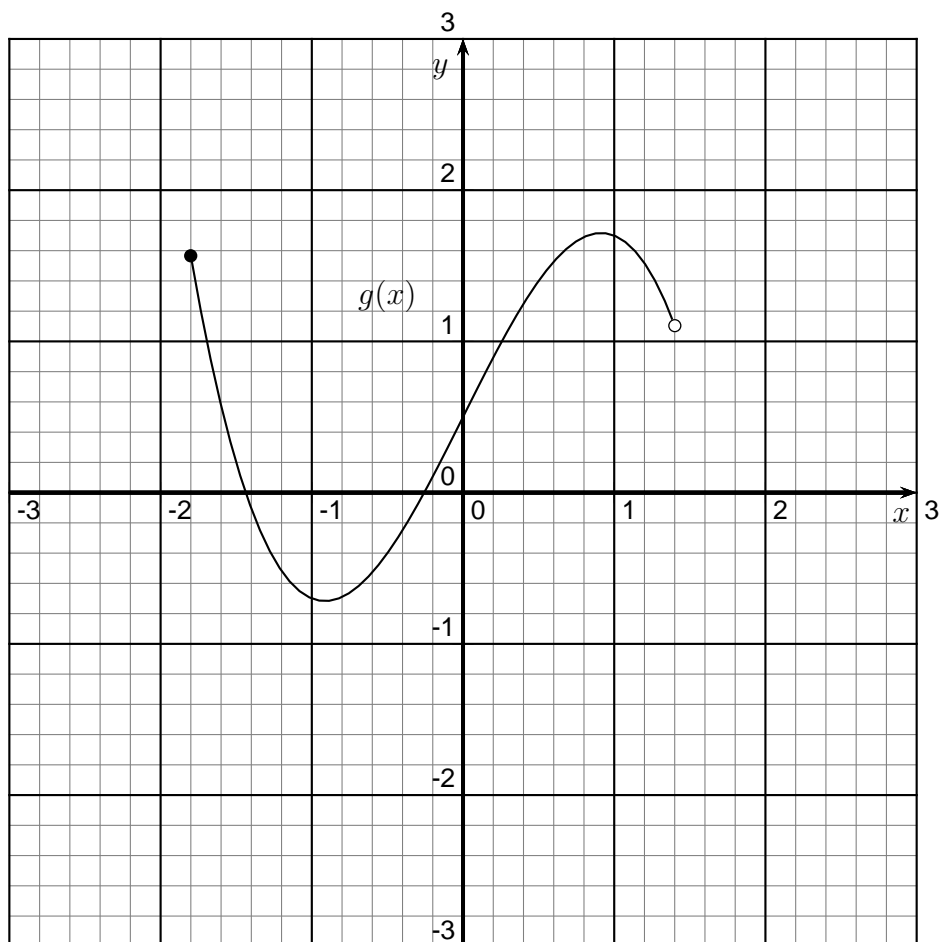
Preliminary Exam

Student Name: _____

Student ID#: _____

Each problem is worth 2 points. Give a complete solution to receive the full credit!

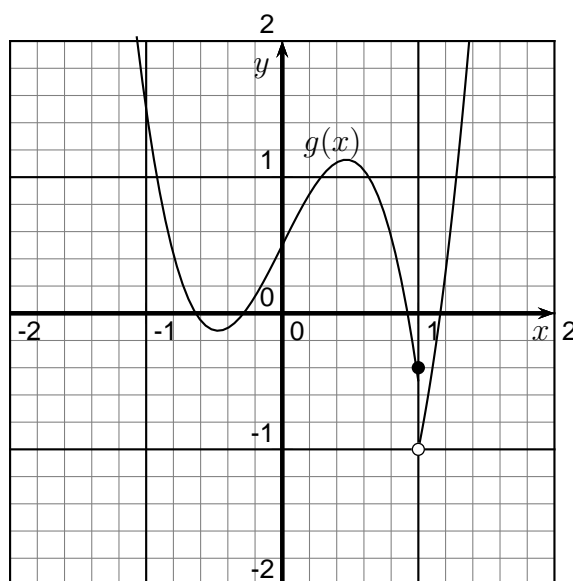
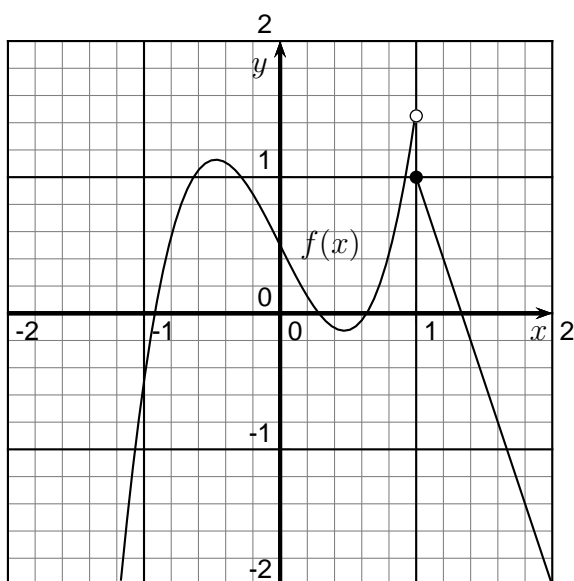
1. The graph of the function $g(x)$ is given.



- (a) Is function $g(x)$ differentiable or not on the interval $(-1.8, 1.4)$.
- (b) State approximately the interval(s) on which $g(x)$ is concave upward.
- (c) Find approximately maximum and minimum values of the function $g(x)$.

2. Write 7th term of the sequence $a_n = (-1)^n \frac{n^2-6}{n^2+16}$, $n = 1, 2, 3, \dots$. Decide if the sequence is converges or diverges. Is it bounded or unbounded?

3. The graphs of f and g are given.



Use them to evaluate $\lim_{x \rightarrow 1^-} (f * g)(x)$ if it exists.

4. The function f is defined by

$$f(x) = \begin{cases} \sinh x - a, & -2 \leq x < 0 \\ 7 - 2 \sin(x), & 0 \leq x \leq 4 \end{cases}$$

where a is a parameter. Find its value so that the function is continuous at the point $x = 0$.

5. Find the best affine approximation of the function $g(z) = \log_5(1+z)$ at the point $z = 0$. Use it to approximate $\log_5 1.1$. What is the difference between the approximate value and the “*true*” value obtained by a calculator?

6. Which of the following logarithms are defined?

- (a) $\log_{0.1}(\ln 1000000000)$
- (b) $\ln(\log(10^{-7}))$
- (c) $\log_{2012}(3.141592653589793 - \pi)$
- (d) $\log_1 2$
- (e) $\log_{11} 0$

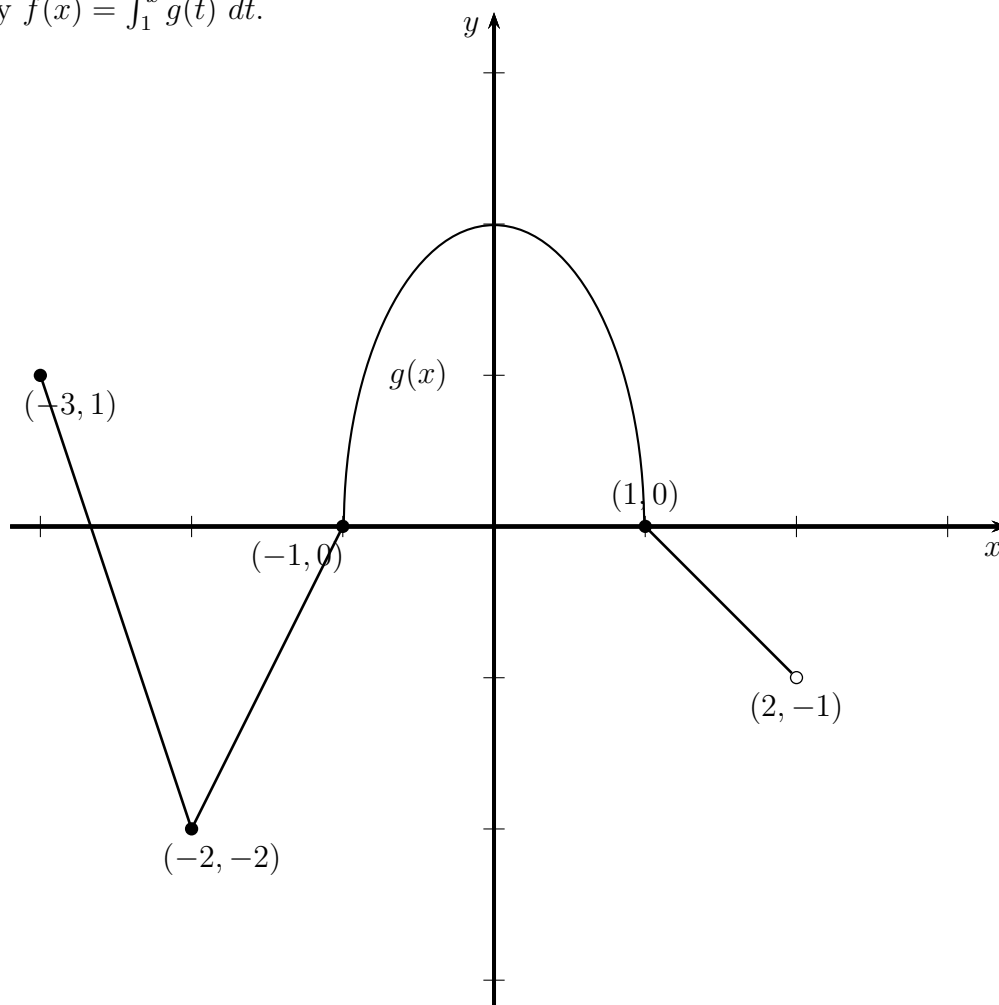
7. Evaluate the integral $\int_{-1}^2 \left(\frac{1}{1-x^2} - 8\sqrt[3]{x^2} \right) dx$.

8. Evaluate

(a) $\int (1 - \cot \theta)^6 \csc^2 \theta \, d\theta.$

(b) $\int (\theta^2 - 1) \cosh(\theta) \, d\theta.$

9. Let g be the continuous function defined on $[-3, 2)$ whose graph, consisting of three line segments and a semiellipse centered at the origin, is given below. Let f be the function given by $f(x) = \int_1^x g(t) dt$.



- (a) Find the values of $f(2)$ and $f(-2)$.
- (b) For each of $f'(-1)$ and $f''(-1)$, find the value or state that it does not exist.

10. Find the derivatives of the following functions.

(a) $f(x) = \left(\arctan\left(\frac{1}{x}\right) - 3\right)^3$

(b) $g(x) = 2^{-\sinh(x)} - \log_5(\ln(x))$