

Exam 1

Student Name: _____
Student ID#: _____

Each problem is worth 5 points. Give a complete solution to receive the full credit!

1. Compute the value of $\sqrt{4.01}$ without calculator using the best linear approximation of the function $f(x) = \sqrt{x}$ around the point 4.

2. Decide whether the function $y(x) = \frac{1}{1+x^2}$ is a solution of the differential equation:

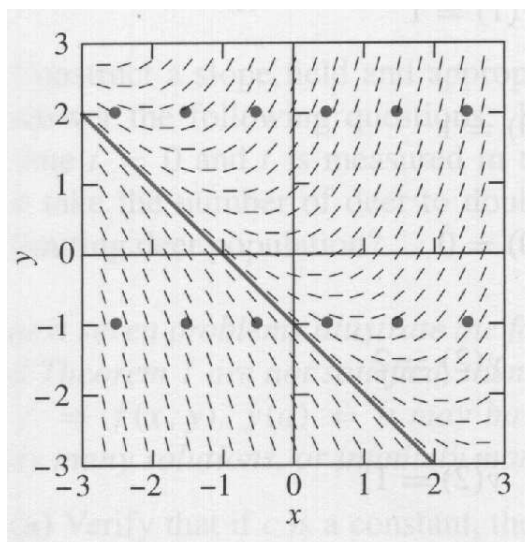
$$y' + 2xy^2 = 0.$$

3. By applying the theorem about the local existence and uniqueness of the solutions of ordinary differential equations verify if the differential equation

$$\frac{dx}{dt} = xe^t$$

has a unique solution satisfying the prescribed initial condition $x(0) = 2e$.

4. The slope field of the differential equation $\frac{dy}{dx} = x + y$ is provided below together with one solution curve. Sketch likely solution curves through the additional points marked in the slope field.



5. Sketch the slope field for the differential equation

$$\dot{x} = t + x$$

6. Find the general solution of the homogeneous linear differential equation of the first order:

$$y' + y \tan x = 0.$$

7. Find the constants A and B such that $y_p = A \sin x + B \cos x$ is a solution of the differential equation:

$$\frac{dy}{dx} + y = 2 \sin x.$$

8. Solve the initial value problem

$$\frac{dy}{dx} + y = 2 \sin x, \quad y(0) = 1.$$

9. Verify if the differential equation

$$(2x + 3y)dx + (3x + 2y)dy = 0$$

is exact.

10. Find the general solution of the differential equation

$$y' = \sqrt{x + y}$$

by any method you know (Hint: Substitution).