Math 3020 Differential Equations
Section A

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Exam 2

Student Name:__________________
Student ID#:__________________

Each problem is worth 5 points. Give a complete solution to receive the full credit!

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1. Consider a linear map $L : \mathbb{R}^2 \to \mathbb{R}^3$. Let

$$B = \left\langle \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\rangle \text{ and } D = \left\langle \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

be bases for $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively. Find the matrix representation $[L]_{B,D}$ of the linear mapping L with respect to the bases **B** and **D** such that

$$L : \begin{pmatrix} 2 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \text{ and } L : \begin{pmatrix} 1 \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}.$$

2. Find the matrix representation $[L]_{\hat{B},D}$ of the linear mapping L from the problem one with respect to the bases $\hat{\mathbf{B}}$ and $\mathbf{D}$ where $\hat{\mathbf{B}}$ is the standard base of $\mathbb{R}^2$ and $\mathbf{D}$ as described in problem one.

3. Write the system

$$\begin{aligned} \dot{x}_1 &= 3x_1 + x_2 \\ \dot{x}_2 &= -x_1 + x_2 \end{aligned} \tag{1}$$

of ODEs in the matrix form.

4. Compute the eigenvalues of the matrix.

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix} \tag{2}$$

5. Is the matrix

$$\hat{A} = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} \tag{3}$$

a simple Jordan block? If it is, write the matrix $\hat{A}$ as the sum of a diagonal matrix and a nilpotent matrix.

6. Compute the eigenvalues of the matrix $\hat{A}$. Are they are the same as the eigenvalues of the matrix A.

7. The system (1) is topologically conjugate to the system.

$$\begin{pmatrix} \dot{\bar{x}}_1 \\ \dot{\bar{x}}_2 \end{pmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \end{pmatrix} \tag{4}$$

Find the fundamental matrix of solutions $\hat{S}$ for system (4).

8. If you knew how to find a change of coordinates M such that

$$\hat{A} = M^{-1}AM$$

you could find the fundamental matrix of solution S for equation (1) as

$$S = M\hat{S}M^{-1}.$$

Since you do not know how to do that I am giving you five free points as long as you sign your name on this spot.

9. Actually, who cares what is the matrix of the fundamental solutions for (1). Qualitatively systems (1) and (4) are identical. Therefore you will get another five points for free as long as you say: "I do not care about special solutions of ODEs, I only care about qualitative behavior".

10. Find the Wronskian of $\hat{S}$.