Math 3280 Linear Algebra
Section A

Instructor: Dr. Predrag Punoševac

Exam 1

Student Name:____________________
Student ID#:____________________

Each problem is worth 5 points. Give a complete solution to receive the full credit!

---

1. Show that $n^2 > 2n$ for $n \geq 3$.

2. Determine if the matrix is singular on non-singular.

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

3. Is the vector $\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$ in the set generated by $\left\{ \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 7 \\ 3 \end{pmatrix} \right\}$.

4. Solve the system

$$\begin{aligned} a + 2b + 3c + d - e &= 1 \\ 3a - b + c + d + e &= 3 \end{aligned}$$

Give the solution in the form of the sum of a particular solution and the solution of the homogeneous part.

5. Represent the vector

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

with respect to the basis

$$B = \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\rangle \subseteq \mathbb{R}^2.$$

6. Decide if the matrices are raw equivalent.

$$\begin{pmatrix} 1 & 1 & 3 \\ 0 & 0 & 3 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 2 \\ 1 & -1 & 1 \end{pmatrix}$$

7. Prove that the set $\mathbb{R}^2$ is a vector space if the operations "+" and "·" have their usual meaning i.e.

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \end{pmatrix}, \quad \lambda \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \lambda \cdot x_1 \\ \lambda \cdot x_2 \end{pmatrix}$$

8. What is the dimension of the span of the set $\{\cos^2\theta, \sin^2\theta, \cos 2\theta, \sin 2\theta\}$

9. Show that any set of four vectors in $\mathbb{R}^2$ is linearly dependent.

10. Find one vector $\vec{v}$ that will make $\langle x, 1 + x^2, \vec{v} \rangle$ into a basis for the space $\mathbb{P}_2$.