Math 3280 Linear Algebra
Section A

Instructor: Dr. Predrag Punoševac

Exam 2

Student Name:__________________
Student ID#:__________________

Each problem is worth 5 points. Give a complete solution to receive the full credit!

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1. Find a basis for the row space of the following matrix.

$$\begin{pmatrix} 2 & 0 & 3 & 4 \\ 3 & 4 & 0 & 2 \\ 1 & 1 & -4 & -2 \end{pmatrix}$$

2. Decide if the vector $(0,0,1)^T$ is in the column space of the matrix.

$$\begin{pmatrix} 1 & 3 & 1 \\ 2 & 0 & 4 \\ 1 & -3 & 3 \end{pmatrix}$$

3. Is the vector $\begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$ in the set generated by $\left\{ \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 7 \\ 3 \end{pmatrix} \right\}$.

4. Determine whether the vectors $\{1 + x, 1 - 2x, x - x^3\}$ are linearly independent in $\mathcal{P}_3$.

5. Decide if the matrices are equivalent.

$$\begin{pmatrix} 1 & 1 & 3 \\ 0 & 1 & 3 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 & 2 \\ 1 & -2 & 1 \end{pmatrix}$$

6. For a linear map from $\mathcal{P}_2$ to $\mathcal{P}_3$ that sends

$$1 \mapsto 1+x, x \mapsto 1-2x, \text{ and } x^2 \mapsto x+x^3$$

where does $1-3x+2x^2$ go?

7. For a linear map from $\mathcal{P}_2$ to $\mathcal{P}_3$ that sends

$$1 \mapsto 1 + x, x \mapsto 1 - 2x, \text{ and } x^2 \mapsto x + x^3$$

find the matrix representation with respect to standard basis for vector spaces $\mathcal{P}_2$ and $\mathcal{P}_3$.

8. Show that $\vec{u}_1 = 1 + x$, $\vec{u}_2 = 1 - 2x$, and $\vec{u}_3 = x^2$ is a base for $\mathcal{P}_2$.

9. Find the transition matrix corresponding to the change of basis from the standard basis of the vector space $\mathcal{P}_2$ to $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ defined in the previous problem.

10. For a linear map from $\mathcal{P}_2$ to $\mathcal{P}_3$ that sends

$$1 \mapsto 1 + x, x \mapsto 1 - 2x, \text{ and } x^2 \mapsto x + x^3$$

find the matrix representation with respect to standard basis for vector spaces $\mathcal{P}_3$ and the basis $\vec{u}_1 = 1 + x$, $\vec{u}_2 = 1 - 2x$, and $\vec{u}_3 = x^2$ of the space $\mathcal{P}_2$.